

0.1 113. Hausaufgabe

0.1.1 Analysis-Buch Seite 256, Aufgabe 15

$$\begin{aligned} \mathbf{a)} \quad \int \sin^2 x \, dx &= \int \sin x (-\cos x)' \, dx = -\sin x \cos x + \int \underbrace{\cos^2 x}_{1-\sin^2 x} \, dx = -\sin x \cos x + x - \int \sin^2 x \, dx; \\ &\Leftrightarrow \int \sin^2 x \, dx = \frac{1}{2} (x - \sin x \cos x); \\ \int_0^\pi \sin^2 x \, dx &= \frac{1}{2} [x - \sin x \cos x]_0^\pi = \frac{\pi}{2}; \end{aligned}$$

$$\mathbf{b)} \quad \int_1^e x \ln x \, dx = \int_1^e \left(\frac{1}{2}x^2\right)' \cdot \ln x \, dx = \left[\frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{x} \cdot \frac{1}{2}x^2 \, dx\right]_1^e = \left[\frac{1}{2}x^2 \cdot \ln x - \frac{1}{4}x^2\right]_1^e = \frac{e^2}{4} + \frac{1}{4};$$

$$\mathbf{c)} \quad \int_1^{e^2} \sqrt{x} \ln x \, dx = \int_1^{e^2} \left(\frac{2}{3}x^{3/2}\right)' \cdot \ln x \, dx = \left[\frac{2}{3}x^{3/2} \cdot \ln x - \int \frac{1}{x} \cdot \frac{2}{3}x^{3/2} \, dx\right]_1^{e^2} = \frac{2}{3} \left[x^{3/2} \cdot \ln x - \frac{2}{3}x^{3/2}\right]_1^{e^2} = \frac{8}{9}e^3 + \frac{4}{9};$$

$$\mathbf{d)} \quad \int_{\sqrt{e}}^e \ln^2 x \, dx = \int_{\sqrt{e}}^e x' \cdot \ln^2 x \, dx = \left[x \cdot \ln^2 x - \int x \cdot 2 \ln x \cdot \frac{1}{x} \, dx\right]_{\sqrt{e}}^e = \left[x \cdot \ln^2 x - 2 \int x \ln x \, dx\right]_{\sqrt{e}}^e = \left[x \cdot \ln^2 x - 2(x \cdot \ln x - x)\right]_{\sqrt{e}}^e = \left[x \cdot (\ln^2 x - 2 \ln x + 2)\right]_{\sqrt{e}}^e = e - \frac{5}{4}\sqrt{e};$$