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0.1 114. Hausaufgabe

0.1.1 Analysis-Buch Seite 256, Aufgabe 14a

$\int e^x \sin x \,\mathrm{d}x = e^x \sin x - \int e^x \cos x \,\mathrm{d}x = e^x \sin x - e^x \cos x - \int e^x \sin x \,\mathrm{d}x; \Leftrightarrow$
$\int e^x \sin x \,\mathrm{d}x = \frac{e^x}{2} (\sin x - \cos x);$

$\int\limits_0^{\pi/2} e^x \sin x \,\mathrm{d}x = \frac{1}{2} \left(e^{\pi/2} + 1\right);$

0.1.2 Analysis-Buch Seite 256, Aufgabe 15

e) $\int\limits_{-1}^{1} \ln x^2 \,\mathrm{d}x = 2 \int\limits_{0}^{1} \ln x^2 \,\mathrm{d}x = 2 \int\limits_{0}^{1} x' {\cdot} \ln x^2 \,\mathrm{d}x = \lim\limits_{\alpha\to 0+} 2 \left[x \ln x^2 - \int \underbrace{x \cdot \frac{1}{x^2} \cdot 2x}_{2} \,\mathrm{d}x \right]_\alpha^1 =$
$\lim\limits_{\alpha\to 0+} 2 \left[x \ln x^2 - 2x \right]_\alpha^1 = -4;$

f) $\int\limits_{1}^{\sqrt{2}} x \ln\left(1 + x^2\right) \,\mathrm{d}x = \int\limits_{1}^{\sqrt{2}} \left(\frac{1}{2} x^2\right)' \ln\left(1 + x^2\right) \,\mathrm{d}x = \left[\frac{1}{2} x^2 \ln\left(1 + x^2\right) - \int \underbrace{\frac{1}{2} x^2 \cdot \frac{1}{1 + x^2} \cdot 2x}_{\frac{x^3}{1+x^2}} \,\mathrm{d}x \right]_1^{\sqrt{2}} =$
$\left[\frac{1}{2} x^2 \ln\left(1 + x^2\right) - \int x \,\mathrm{d}x - \underbrace{\frac{x}{1 + x^2}}_{\frac{(1+x^2)'}{1+x^2} \cdot \frac{1}{2}} \,\mathrm{d}x \right]_1^{\sqrt{2}} = \left[\frac{1}{2} x^2 \ln\left(1 + x^2\right) - \frac{1}{2} x^2 + \frac{1}{2} \ln\left|1 + x^2\right| \right]_1^{\sqrt{2}} =$
$\frac{1}{2} \left[\left(1 + x^2\right) \ln\left(1 + x^2\right) - x^2 \right]_1^{\sqrt{2}} = \frac{1}{2} \left(3 \ln 3 - 2 \ln 2 - 1\right);$

g) $\int\limits_{0}^{1} x^{-1/2} \ln x \,\mathrm{d}x = \int\limits_{0}^{1} \left(2x^{1/2}\right)' \ln x \,\mathrm{d}x = \lim\limits_{\alpha\to 0+} \left[2\sqrt{x} \ln x - \int 2 \underbrace{x^{1/2} x^{-1}}_{x^{-1/2}} \,\mathrm{d}x \right]_\alpha^1 = \lim\limits_{\alpha\to 0+} \left[2\sqrt{x} \left(\ln x - 2\right) \right]_\alpha^1 =$
$-4;$

0.1.3 Analysis-Buch Seite 256, Aufgabe 17a

Zeige, dass gilt:

$$\int \sin^n x \, \mathrm{d}x = -\frac{1}{n} (\sin x)^{n-1} \cos x + \frac{n-1}{n} \int (\sin x)^{n-2} \, \mathrm{d}x;$$

$$\left[-\tfrac{1}{n} (\sin x)^{n-1} \cos x + \tfrac{n-1}{n} \textstyle\int (\sin x)^{n-2} \, \mathrm{d}x\right]' =$$

$$= \tfrac{1}{n} \left[(\sin x)^{n-1} \sin x - (n-1) (\sin x)^{n-2} \underbrace{\cos^2 x}_{1-\sin^2 x} + (n-1) (\sin x)^{n-2} \right] =$$

$$= \tfrac{1}{n} \sin^n x \left[1 - (n-1) (\sin x)^{-2} \left(1 - \sin^2 x - 1\right)\right] =$$

$$= \tfrac{1}{n} \sin^n x \cdot (1 + n - 1) = \sin^n x;$$