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0.1 4. Hausaufgabe

0.1.1 Analysis-Buch Seite 15, Aufgabe 9

Zeige die Richtigkeit von

a) $\int (x^2 - x)\,\mathrm{d}x = \frac{1}{3}x^3 - \frac{1}{2}x^2 + C$;
$\left(\frac{1}{3}x^3 - \frac{1}{2}x^2 + C\right)' = x^2 - x$;

b) $\int \sqrt{x}\,\mathrm{d}x = \frac{2}{3}x\sqrt{x} + C$;
$\left(\frac{2}{3}x\sqrt{x} + C\right)' = \left(\frac{2}{3}x^{\frac{3}{2}} + C\right)' = x^{\frac{1}{2}} = \sqrt{x}$;

c) $\int \frac{1}{x^2}\,\mathrm{d}x = -\frac{1}{x} + C$;
$\left(-\frac{1}{x} + C\right)' = \frac{1}{2}x^2$;

d) $\int \sin^2 x\,\mathrm{d}x = \frac{1}{2}\left(x - \sin x \cos x\right) + C$;
$\left[\frac{1}{2}\left(x - \sin x \cos x\right) + C\right]' = \frac{1}{2}\left(1 - \cos x \cos x + \sin x \sin x\right) = \frac{1}{2}\left(1 - 1 + \sin^2 x + \sin^2 x\right) = \sin^2 x$;

e) $\int \cos^2 x\,\mathrm{d}x = \frac{1}{2}\left(x + \sin x \cos x\right) + C$;
$\left[\frac{1}{2}\left(x + \sin x \cos x\right) + C\right]' = \frac{1}{2}\left(1 + \cos x \cos x - \sin x \sin x\right) = \frac{1}{2}\left(1 - 1 + \cos^2 x + \cos^2 x\right) = \cos^2 x$;

f) $\int \frac{x}{\sqrt{a^2 - x^2}}\,\mathrm{d}x = -\sqrt{a^2 - x^2} + C$;
$\left(-\sqrt{a^2 - x^2} + C\right)' = -\dfrac{1}{2\sqrt{a^2 - x^2}}\left(0 - 2x\right) = \dfrac{x}{\sqrt{a^2 - x^2}}$;