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0.1 77. Hausaufgabe

0.1.1 Geometrie-Buch Seite 117, Aufgabe 2

Im Dreieck ABC ist $\overrightarrow{BD} = \frac{3}{4}\overrightarrow{BD}$ und $\overrightarrow{AS} = \frac{1}{2}\overrightarrow{AD}$. BS schneidet AC in T.

In welchem Verhältnis teilt T die Strecke $\overrightarrow{AC}$ beziehungsweise S die Strecke $\overrightarrow{BT}$?

$\overrightarrow{AB}$, $\overrightarrow{AC}$ linear unabhängig.

$$\overrightarrow{AS} + \overrightarrow{ST} + \overrightarrow{TA} =$$
$$= \frac{1}{2}\underbrace{\left(\overrightarrow{AB} + \frac{3}{4}\overrightarrow{BC}\right)}_{\overrightarrow{AD}} + \underbrace{\lambda\overrightarrow{BT}}_{\overrightarrow{ST}} + \underbrace{\mu\overrightarrow{CA}}_{\overrightarrow{TA}} =$$
$$= \frac{1}{2}\overrightarrow{AB} + \frac{3}{8}\underbrace{\left(\overrightarrow{BA} + \overrightarrow{AC}\right)}_{\overrightarrow{BC}} + \lambda\underbrace{\left(\overrightarrow{BA} - \mu\overrightarrow{CA}\right)}_{\overrightarrow{BT}} + \mu\overrightarrow{CA} =$$
$$= \overrightarrow{AB}\left(\frac{1}{2} - \frac{3}{8} - \lambda\right) + \overrightarrow{AC}\left(\frac{3}{8} + \lambda\mu - \mu\right) =$$
$$= \vec{0};$$

$\lambda = \frac{1}{2} - \frac{3}{8} = \frac{1}{8};$

$\mu = \frac{\frac{3}{8}}{1-\lambda} = \frac{3}{7};$

$\overrightarrow{BS} = \beta\overrightarrow{ST}; \Leftrightarrow \beta = \frac{\overrightarrow{BS}}{\overrightarrow{ST}} = \frac{\overrightarrow{BT}+\overrightarrow{TS}}{\lambda\overrightarrow{BT}} = \frac{\overrightarrow{BT}-\lambda\overrightarrow{BT}}{\lambda\overrightarrow{BT}} = \frac{1-\lambda}{\lambda} = 7;$

$\overrightarrow{AT} = \alpha\overrightarrow{TC}; \Leftrightarrow \alpha = \frac{\overrightarrow{AT}}{\overrightarrow{TC}} = \frac{\overrightarrow{AT}}{\overrightarrow{TA}+\overrightarrow{AC}} = \frac{\mu\overrightarrow{AC}}{-\mu\overrightarrow{AC}+\overrightarrow{AC}} = \frac{\mu}{1-\mu} = \frac{3}{4};$