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## 0.1 8. Hausaufgabe

### 0.1.1 Analysis-Buch Seite 36, Aufgabe 17a

Berechne mit Hilfe der Streifenmetnode den Flächeninhalt von

$F := \{(x,y)\,|\,0 \le x \le 1 \wedge 0 \le y \le 1 - x^2\}$ ;

$[\sum_{i=1}^{n} i = \frac{n(n+1)}{2}; \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6};]$

$\mathrm{f}(x) = 1 - x^2;$

$$\begin{aligned} S_n &= \sum_{i=1}^{n} \frac{1}{n}\mathrm{f}\left(\frac{i-1}{n}\right) = \frac{1}{n}\sum_{i=1}^{n}\left[1 - \frac{i^2-2i+1}{n^2}\right] = \frac{n\cdot 1}{n} + \frac{1}{n}\sum_{i=1}^{n} -\frac{i^2-2i+1}{n^2} = \\ &= 1 - \frac{1}{n^3}\sum_{i=1}^{n}\left[i^2 - 2i + 1\right] = 1 - \frac{1}{n^3}\left(\sum_{i=1}^{n} i^2 + 2\sum_{i=1}^{n} i + n\right) = \\ &= 1 - \frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6} + n\,(n+1) + n\right] = 1 - \frac{2n^2+9n+13}{6n^2}; \end{aligned}$$

$$\begin{aligned} s_n &= \sum_{i=1}^{n} \frac{1}{n}\mathrm{f}\left(\frac{i}{n}\right) = \frac{1}{n}\sum_{i=1}^{n}\left[1 - \frac{i^2}{n^2}\right] = \frac{n\cdot 1}{n} - \frac{1}{n^3}\sum_{i=1}^{n} i^2 = \\ &= 1 - \frac{n(n+1)(2n+1)}{6n^3} = 1 - \frac{2n^2+n+2n+1}{6n^2} = 1 - \frac{2n^2+3n+1}{6n^2}; \end{aligned}$$

$$\left.\begin{array}{l} \lim_{n\to\infty} S_n = 1 - \frac{1}{3} = \frac{2}{3}; \\ \lim_{n\to\infty} s_n = 1 - \frac{1}{3} = \frac{2}{3}; \end{array}\right\} \Rightarrow \lim_{n\to\infty} S_n = \lim_{n\to\infty} s_n = \frac{2}{3} =: A;$$