0.1 83. Hausaufgabe

0.1.1 Analysis-Buch Seite 115, Aufgabe 61

Berechne:

a) $\lim\limits_{x\to\infty}\left(\frac{2x+1}{2x-1}\right)^{2x} = \lim\limits_{x\to\infty}\left(\frac{2x-1+1+1}{2x-1}\right)^{2x} = \lim\limits_{x\to\infty}\left(1+\frac{2}{2x-1}\right)^{2x} = \lim\limits_{x\to\infty}\left(1+\frac{2}{2x-1}\right)^{2x-1+1} =$
$\lim\limits_{u\to\infty}\left(1+\frac{2}{u}\right)^{u+1} = \lim\limits_{u\to\infty}\left(1+\frac{2}{u}\right)^{u}\cdot\left(1+\frac{2}{u}\right) = e^2\cdot 1 = e^2;$

b) $\lim\limits_{x\to\infty}\left(\frac{2x+1}{3x-1}\right)^{2x} = \lim\limits_{x\to\infty}\left(\frac{2}{3}\right)^{2x} = 0;$

c) $\lim\limits_{x\to\infty}\left(\frac{3x+1}{2x-1}\right)^{2x} = \lim\limits_{x\to\infty}\left(\frac{3}{2}\right)^{2x} = \infty;$

d) $\lim\limits_{x\to\infty}\left(\frac{3x+1}{3x-1}\right)^{2x} = \lim\limits_{x\to\infty}\left(\frac{3x-1+1+1}{3x-1}\right)^{2x} = \lim\limits_{x\to\infty}\left(1+\frac{2}{3x-1}\right)^{2x} = \lim\limits_{x\to\infty}\left(1+\frac{2}{3x-1}\right)^{2x+x-x-1+1} =$
$\lim\limits_{x\to\infty}\left(1+\frac{2}{3x-1}\right)^{(3x-1)\cdot\frac{2}{3}+\frac{2}{3}} = \left(e^2\right)^{\frac{2}{3}} = e^{\frac{4}{3}};$

e) $\lim\limits_{x\to\infty}\left(\frac{2x+2}{2x-1}\right)^{2x} = \lim\limits_{x\to\infty}\left(\frac{2x-1+1+2}{2x-1}\right)^{2x} = \lim\limits_{x\to\infty}\left(1+\frac{3}{2x-1}\right)^{2x} = \lim\limits_{x\to\infty}\left[\left(1+\frac{3/2}{x}\right)^{x}\right]^{2} =$
$\left(e^{\frac{3}{2}}\right)^{2} = e^3;$

f) $\lim\limits_{x\to\infty}\left(\frac{2x+2}{2x-2}\right)^{2x} = \lim\limits_{x\to\infty}\left(\frac{2x-2+2+2}{2x-2}\right)^{2x} = \lim\limits_{x\to\infty}\left(1+\frac{4}{2x-1}\right)^{2x} = \lim\limits_{x\to\infty}\left[\left(1+\frac{4/2}{x}\right)^{x}\right]^{2} =$
$\left(e^{\frac{4}{2}}\right)^{2} = e^4;$

XXX „Plusminus 1 wird bei Unendlich schon nichts ausmachen"
nicht sehr elegant (Aufgaben e) und f))