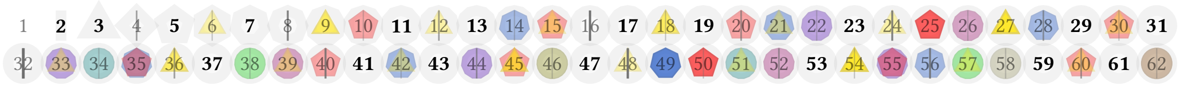

Large numbers, very large numbers, very very large numbers

– an invitation to advanced googology –

Ingo Blechschmidt

37th Chaos Communication Congress
December 30th, 2023

Part 0

Large numbers

12 000 congress participants

$10^{19} = \underbrace{10\,000\,000\,000\,000\,000\,000}_{\text{19 zeros}}$ grains of sand on earth

$10^{80} = 1\underbrace{000\ldots000}_{\text{80 zeros}}$ elementary particles in the universe

Part I

Very large numbers

$$\begin{aligned} 2 \cdot 4 &= 2 + 2 + 2 + 2 = 8 \\ 2^4 &= 2 \cdot 2 \cdot 2 \cdot 2 = 16 \end{aligned}$$

Part I

Very large numbers

$$2 \cdot 4 = 2 + 2 + 2 + 2 = 8$$

$$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$$

$$2 \uparrow\uparrow 4 = 2^{2^{2^{2}}} = 2^{2^{4}} = 2^{16} = 65\,536$$

Part I

Very large numbers

$$
\begin{aligned}
2 \cdot 4 &= 2 + 2 + 2 + 2 = 8 \\
2^4 &= 2 \cdot 2 \cdot 2 \cdot 2 = 16 \\
2 \uparrow\uparrow 4 &= 2^{2^{2^2}} = 2^{2^4} = 2^{16} = 65\,536 \\
2 \uparrow\uparrow\uparrow 4 &= 2 \uparrow\uparrow (2 \uparrow\uparrow (2 \uparrow\uparrow 2))
\end{aligned}
$$

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$$= \left.2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}\right\}\text{65 536 many two's}$$

Part I

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Part I
Very large numbers

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$$= 2 \uparrow\uparrow\uparrow 2^{2^{\cdot^{\cdot^{\cdot^{2}}}}} = \underbrace{2 \uparrow\uparrow (2 \uparrow\uparrow (2 \uparrow\uparrow (\cdots \uparrow\uparrow 2)))}_{2^{2^{\cdot^{\cdot^{\cdot^{2}}}}} \text{ many two's}}$$

Part I

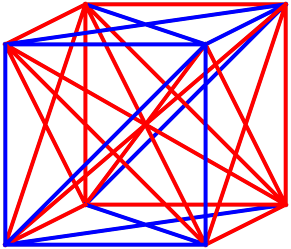
Very large numbers

Graham's number $= \left.\begin{array}{c} 3\uparrow \cdots\cdots\cdots\cdots \uparrow 3 \\ \underbrace{\qquad\qquad\qquad\qquad}_{} \\ 3\uparrow \cdots\cdots\cdots \uparrow 3 \\ \underbrace{\qquad\qquad\qquad}_{} \\ \vdots \\ \underbrace{\qquad\qquad\qquad}_{} \\ 3\uparrow \cdots\cdots \uparrow 3 \\ \underbrace{\qquad\qquad}_{} \\ 3\uparrow\uparrow\uparrow\uparrow 3 \end{array}\right\} \text{64 layers}$

Part I

Very large numbers

$$\text{Graham's number} = \left.\begin{matrix} 3\underbrace{\uparrow \cdots\cdots\cdots\cdots\cdots\cdots \uparrow}3 \\ 3\underbrace{\uparrow \cdots\cdots\cdots\cdots\cdots \uparrow}3 \\ \vdots \\ \underbrace{\qquad\qquad\qquad\qquad} \\ 3\underbrace{\uparrow \cdots\cdots \uparrow}3 \\ 3\uparrow\uparrow\uparrow\uparrow 3 \end{matrix}\right\} \text{64 layers}$$



$$\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdot^{\cdot^{\cdot}}}}}} = 2$$

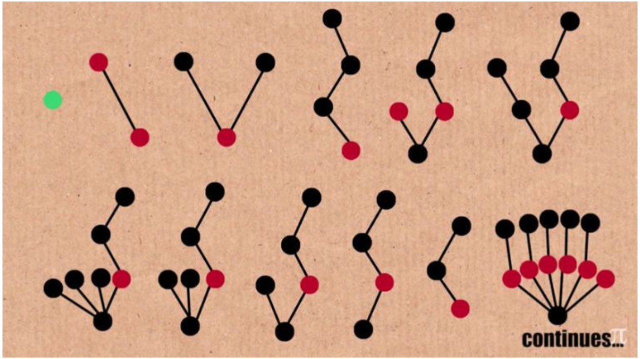
$$\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdot^{\cdot^{\cdot}}}}}} = 2$$

$$\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}} = 2$$

$$\frac{2}{\pi} = \sqrt{\frac{1}{2}} \cdot \sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}} \cdot \sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}} \cdot \ldots$$

Part II

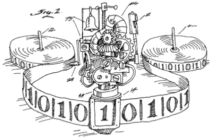
Very very large numbers



Every forest eventually repeats, at a maximum of **TREE(3)** trees.

Part III

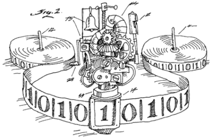
Very very very large numbers



- **BB(n)** is the maximal number of steps a Turing machine with n states can carry out before halting.

Part III

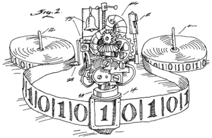
Very very very large numbers



- **BB(n)** is the maximal number of steps a Turing machine with n states can carry out before halting.
- The Busy Beaver function is **uncomputable**.

Part III

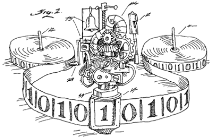
Very very very large numbers



- **BB(n)** is the maximal number of steps a Turing machine with n states can carry out before halting.
- The Busy Beaver function is **uncomputable** and **asymptotically dominates** any computable function.

Part III

Very very very large numbers



- **BB(n)** is the maximal number of steps a Turing machine with n states can carry out before halting.
- The Busy Beaver function is **uncomputable** and **asymptotically dominates** any computable function.
- **Provably so,** no conjecture regarding the value of BB(748) is provable, not even "BB(748) = ♡" where ♡ is the true value of BB(748).

Part V

Very very very very large numbers

- **Rayo(n)** is the largest natural number uniquely definable using n symbols in the mathematical language of ZFC.
- The Rayo function is **(ZFC-)undefinable** and **asymptotically dominates** any (ZFC-)definable function.

Award ceremony

category	number of submissions
disqualified	1
small-on-purpose	3
physical	3
nines	5
hyper	5
TREE	1
Busy Beaver	0
Rayo	7
referential	17
fun	1

46 submissions

42 • Bla • brodo • Chenjox • Claire • Daniel • Dikshita Kalita • Domino • dreieck • esclear • Henning • j4riO • Jannis • Kai • Kampfzwerg • Laura mit den roten Haaren • lennard, gerrit, mirko • luap42 • maksio • mob • pacey • Pauchu • pskirde • polyma3000 • ricl • SAFT • smyds • T • [this field intentionally left blank] • timo • timo2 • ves • xopn • Yorick • Yorick, Mickey