2008 / Mechanik I (Reflexion an weicher Wand)

$$V(x) = \frac{V_0}{2} e^{-\alpha x} \qquad \alpha > 0, \ V_0 > 0$$

a) $E = \frac{1}{2} m v^2 + \frac{V_0}{2} e^{-\alpha x}$

$v(x=\infty) = v_\infty \quad \Rightarrow \quad E_0 = \frac{1}{2} m v_\infty^2 = \frac{V_0}{2} e^{-\alpha x_0}$

Umkehrpunkt x_0: $v = 0$

$$\frac{1}{2} m v_\infty^2 = \frac{V_0}{2} e^{-\alpha x_0}$$

$$\frac{m v_\infty^2}{V_0} = e^{-\alpha x_0}$$

$$x_0 = -\frac{1}{\alpha} \ln\left(\frac{m v_\infty^2}{V_0}\right)$$

b) $x(0) = 0$

$$\frac{1}{2} m \dot{x}^2 + \frac{V_0}{2} e^{-\alpha x} = E$$

$$\frac{dx}{dt} = \dot{x} = \sqrt{\frac{1}{m}\left(2E_0 - V_0 e^{-\alpha x}\right)}$$

$$\int_0^t dt' = t = \int_{x_0}^{x} dx' \frac{\sqrt{m}}{\sqrt{2E_0 - V_0 e^{-\alpha x'}}}$$

$$= \int_{x_0}^{x} dx' \sqrt{\frac{m}{2E_0}} \sqrt{\frac{1}{1 - \frac{V_0 e^{-\alpha x'}}{2E_0}}} = \int_{x_0}^{x} dx' \sqrt{\frac{m}{2E_0}} \sqrt{\frac{1}{1 - e^{-\alpha(x'-x_0)}}}$$

$s = x' - x_0 \qquad x' = s + x_0$

$ds = dx'$

$$= \sqrt{\frac{m}{2E_0}} \int_0^{s-x_0} ds \frac{1}{\sqrt{1 - e^{-\alpha s}}} = \sqrt{\frac{m}{2E_0}} \frac{2}{\alpha} \operatorname{acosh}\left(e^{\frac{\alpha}{2}(x-x_0)}\right)$$

$$\ln\left[\cosh\left(\frac{\alpha t}{2}\sqrt{\frac{2E_0}{m}}\right)\right]\frac{2}{\alpha} + x_0 = x(t) \qquad x(t) = \ln\cosh(kt) \ \text{mit} \ k = \frac{\alpha}{2}\sqrt{\frac{2E_0}{m}} > 0$$

c) $$\dot{x}(t) = \frac{2}{\alpha} \frac{\sinh\left(\frac{\alpha t}{2}\sqrt{\frac{2E_0}{m}}\right)}{\cosh\left(\frac{\alpha t}{2}\sqrt{\frac{2E_0}{m}}\right)} \frac{\alpha}{2}\sqrt{\frac{2E_0}{m}} = \sqrt{\frac{2E_0}{m}} \tanh\left(\frac{\alpha t}{2}\sqrt{\frac{2E_0}{m}}\right)$$

$$= \sqrt{\frac{2E_0}{m}} \tanh(kt)$$

$$\cosh x = e^{x} + e^{-} \approx \begin{cases} e^{x} & x >> 1 \\ e^{-x} & x << 1 \end{cases}$$

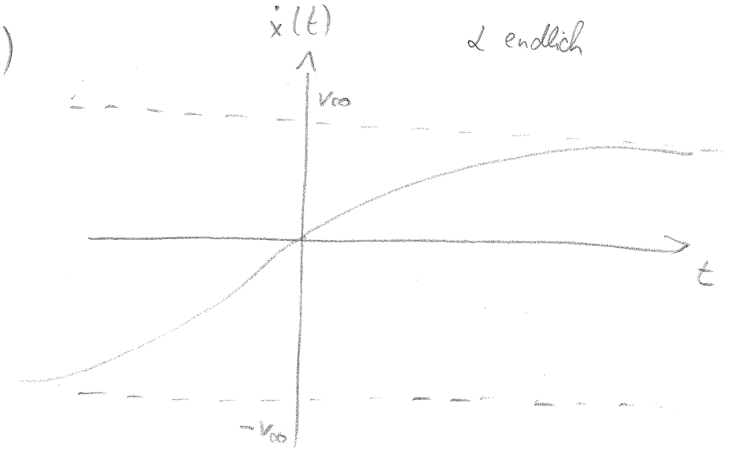
$$\lim_{t \to +\infty} x(t) = \lim_{t \to \infty} \ln \cosh(kt) + x_0 = \lim_{t \to \infty} \left(\ln \left[e^{kt} + e^{-kt} \right] + x_0 \right)$$

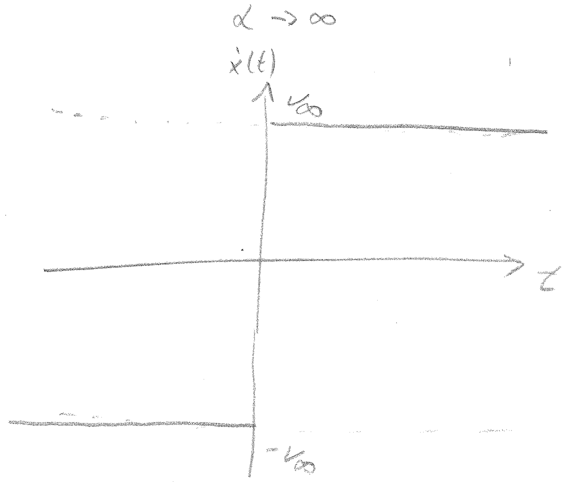
$$= \lim_{t \to \infty} \left(kt + x_0 \right) = \infty$$

$$\lim_{t \to -\infty} x(t) = +\infty$$

$$\lim_{t \to \pm\infty} \dot{x}(t) = \pm \sqrt{\frac{2E_0}{m}} = \pm v_\infty$$

d)





$\Rightarrow$ harte Wand